

THE DYNKIN DIAGRAMS PACKAGE

VERSION 3.141

BEN MCKAY

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1. QUICK INTRODUCTION

See section 24 for the latest changes to earlier versions.

Load the Dynkin diagram package (see options below)

```
\documentclass{amsart}
\usepackage{dynkin-diagrams}
\begin{document}
The Dynkin diagram of  $(B_3)$  is  $\text{dynkin}\{B\}\{3\}$ .
\end{document}
```

Invoke it

The Dynkin diagram of $\backslash(B_3\backslash)$ is `\dynkin{B}{3}`.

The Dynkin diagram of B_3 is $\bullet \bullet \rightarrow \bullet$.

Inside a *TikZ* statement

The Dynkin diagram of $\backslash(B_3\backslash)$ is
`\tikz \dynkin{B}{3};`

The Dynkin diagram of B_3 is $\bullet \bullet \rightarrow \bullet$

Inside a Dynkin diagram environment

The Dynkin diagram of $\backslash(B_3\backslash)$ is
`\begin{dynkinDiagram}{B}{3}`
`\draw[very thick,red] (root 1) to [out=-45, in=-135] (root 3);`
`\end{dynkinDiagram}`

The Dynkin diagram of B_3 is $\bullet \bullet \rightarrow \bullet$

Inside a *TikZ* environment

The baseline controls the vertical alignment:
the Dynkin diagram of $\backslash(B_3\backslash)$ is
`\begin{tikzpicture}[baseline=(origin.base)]`
`\dynkin{B}{3}`
`\draw[very thick,red] (root 1) to [out=-45, in=-135] (root 3);`
`\end{tikzpicture}`

The baseline controls the vertical alignment: the Dynkin diagram of B_3 is



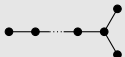


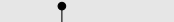




Indefinite rank Dynkin diagrams

`\dynkin{B}{}`



Table 1: The Dynkin diagrams of the reduced simple root systems
[3] pp. 265–290, plates I–IX

A_n		<code>\dynkin{A}{}</code>
C_n		<code>\dynkin{C}{}</code>
D_n		<code>\dynkin{D}{}</code>
E_6		<code>\dynkin{E}{6}</code>
E_7		<code>\dynkin{E}{7}</code>
E_8		<code>\dynkin{E}{8}</code>
F_4		<code>\dynkin{F}{4}</code>
G_2		<code>\dynkin{G}{2}</code>

2. SET OPTIONS GLOBALLY

Most options set globally ...

```
\pgfkeys{/Dynkin diagram,edge length=.5cm,fold radius=.5cm}
```

...or pass global options to the package

```
\usepackage[
  ordering=Kac,
  edge/.style=blue,
  mark=o,
  root radius=.06cm]
{dynkin-diagrams}
```

3. COXETER DIAGRAMS

Coxeter diagram option

```
\dynkin[Coxeter]{F}{4}
```



gonality option for G_2 and I_n Coxeter diagrams

```
\(G_2=\dynkin[Coxeter,gonality=n]{G}{2}\), \
\ (I_n=\dynkin[Coxeter,gonality=n]{I}{})
```

$$G_2 = \overset{n}{\bullet} \text{---} \bullet, \quad I_n = \bullet \text{---} \overset{n}{\bullet}$$

Table 2: The Coxeter diagrams of the simple reflection groups

A_n		<code>\dynkin[Coxeter]{A}{}</code>
B_n		<code>\dynkin[Coxeter]{B}{}</code>
C_n		<code>\dynkin[Coxeter]{C}{}</code>
E_6		<code>\dynkin[Coxeter]{E}{6}</code>
E_7		<code>\dynkin[Coxeter]{E}{7}</code>
E_8		<code>\dynkin[Coxeter]{E}{8}</code>
F_4		<code>\dynkin[Coxeter]{F}{4}</code>
G_2		<code>\dynkin[Coxeter,gonality=n]{G}{2}</code>
H_3		<code>\dynkin[Coxeter]{H}{3}</code>
H_4		<code>\dynkin[Coxeter]{H}{4}</code>
I_n		<code>\dynkin[Coxeter,gonality=n]{I}{}</code>

4. SATAKE DIAGRAMS

Satake diagrams use the standard name instead of a rank

`\(A_{IIIb}=\dynkin{A}{IIIb}\)`

$$A_{IIIb} = \text{Diagram with 4 nodes in a square, with a double arrow on the right edge.}$$

We use a solid gray bar to denote the folding of a Dynkin diagram, rather than the usual double arrow, since the diagrams turn out simpler and easier to read.

Table 3: The Satake diagrams of the real simple Lie algebras [13] p. 532–534

A_I		<code>\dynkin{A}{I}</code>
A_{II}		<code>\dynkin{A}{II}</code>
A_{IIIa}		<code>\dynkin{A}{IIIa}</code>
A_{IIIb}		<code>\dynkin{A}{IIIb}</code>

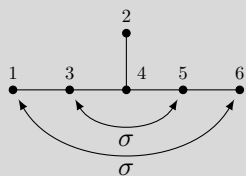
continued ...

Table 3: ...continued

A_{IV}		<code>\dynkin{A}{IV}</code>
B_I		<code>\dynkin{B}{I}</code>
B_{II}		<code>\dynkin{B}{II}</code>
C_I		<code>\dynkin{C}{I}</code>
C_{IIa}		<code>\dynkin{C}{IIa}</code>
C_{IIb}		<code>\dynkin{C}{IIb}</code>
D_{Ia}		<code>\dynkin{D}{Ia}</code>
D_{Ib}		<code>\dynkin{D}{Ib}</code>
D_{Ic}		<code>\dynkin{D}{Ic}</code>
D_{II}		<code>\dynkin{D}{II}</code>
D_{IIIa}		<code>\dynkin{D}{IIIa}</code>
D_{IIIb}		<code>\dynkin{D}{IIIb}</code>
E_I		<code>\dynkin{E}{I}</code>
E_{II}		<code>\dynkin{E}{II}</code>
E_{III}		<code>\dynkin{E}{III}</code>
E_{IV}		<code>\dynkin{E}{IV}</code>
E_V		<code>\dynkin{E}{V}</code>
E_{VI}		<code>\dynkin{E}{VI}</code>
E_{VII}		<code>\dynkin{E}{VII}</code>
E_{VIII}		<code>\dynkin{E}{VIII}</code>
E_{IX}		<code>\dynkin{E}{IX}</code>
F_I		<code>\dynkin{F}{I}</code>
F_{II}		<code>\dynkin{F}{II}</code>
G_I		<code>\dynkin{G}{I}</code>

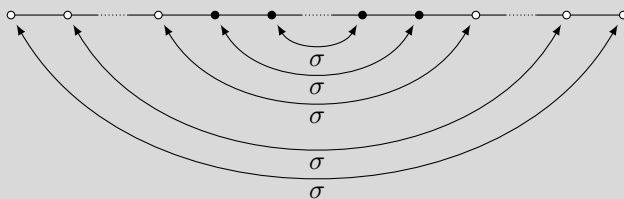
If you don't like the solid gray "folding bar", most people use arrows. Here is E_{II}

```
\newcommand{\invol}[2]{\draw[latex-latex] (root #1) to
[out=-60,in=-120] node[midway,below]{\sigma} (root #2);}
\begin{dynkinDiagram}[edge length=.75cm,labels*={1,\dots,6}]{E}{6}
\invol{1}{6}\invol{3}{5}
\end{dynkinDiagram}
```



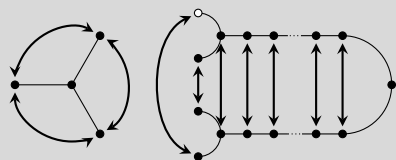
The double arrows for A_{IIIa} are big

```
\newcommand{\invol}[2]{\draw[latex-latex] (root #1) to
[out=-60,in=-120] node[midway,below]{\sigma} (root #2);}
\begin{dynkinDiagram}[edge length=.75cm]{A}{oo.o**.*o.oo}
\invol{1}{10}\invol{2}{9}\invol{3}{8}\invol{4}{7}\invol{5}{6}
\end{dynkinDiagram}
```



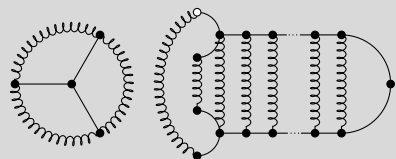
If you don't like the solid gray "folding bar", most people use arrows ...

```
\tikzset{/Dynkin diagram/fold style/.style={stealth-stealth,thick,
shorten <=1mm,shorten >=1mm,}}
\dynkin[ply=3,edge length=.75cm]{D}{4}
\begin{dynkinDiagram}[ply=4]{D}{1}%
{****.*****.*****}
\dynkinFold{1}{13}
\dynkinFold[bend right=65]{0}{14}
\end{dynkinDiagram}
```



...but you could try springs pulling roots together

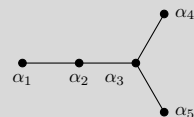
```
\tikzset{Dynkin diagram/fold style/.style=
{decorate,decoration={name=coil,aspect=0.5,
segment length=1mm,amplitude=.6mm}}}
\dynkin[ply=3,edge length=.75cm]{D}{4}
\begin{dynkinDiagram}[ply=4]{D}[1]%
{****.*****.*****}
\dynkinFold{1}{13}
\dynkinFold[bend right=65]{0}{14}
\end{dynkinDiagram}
```



5. LABELS FOR THE ROOTS

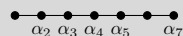
Make a macro to assign labels to roots

```
\dynkin[label,label macro/.code={\alpha_{#1}},edge
length=.75cm]{D}{5}
```



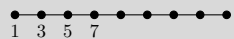
Labelling several roots

```
\dynkin[labels={,2,...,5,,7},label macro/.code={\alpha_{#1}}]{A}{7}
```



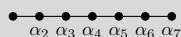
The foreach notation I

```
\dynkin[labels={1,3,...,7},]{A}{9}
```



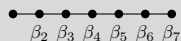
The foreach notation II

```
\dynkin[labels={,\alpha_2,\alpha_...\alpha_7},]{A}{7}
```



The foreach notation III

```
\dynkin[label macro/.code={\beta_{#1}},labels={,2,...,7},]{A}{7}
```



Label the roots individually by root number

```
\dynkin[label]{B}{3}
```



Label a single root

```
\begin{dynkinDiagram}{B}{3}
\dynkinLabelRoot{2}{\alpha_2}
\end{dynkinDiagram}
```



Use a text style

```
\begin{dynkinDiagram}[text/.style={scale=1.2}]{B}{3};
\dynkinLabelRoot{2}{\alpha_2}
\end{dynkinDiagram}
```



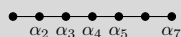
Access root labels via TikZ

```
\begin{dynkinDiagram}{B}{3}
\node[below] at (root 2) {\(\alpha_2\)};
\end{dynkinDiagram}
```



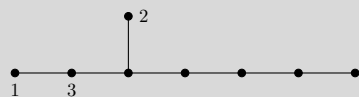
Commands to label several roots

```
\begin{dynkinDiagram}{A}{7}
\dynkinLabelRoots{,\alpha_2,\alpha_3,\alpha_4,\alpha_5,,\alpha_7}
\end{dynkinDiagram}
```



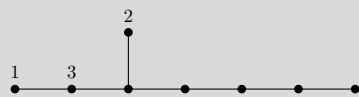
The labels have default locations, mostly below roots

```
\dynkin[edge length=.75cm,labels={1,2,3}]{E}{8}
```



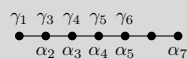
The starred form flips labels to alternate locations, mostly above roots

```
\dynkin[edge length=.75cm,labels*={1,2,3}]{E}{8}
```



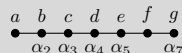
Labelling several roots and alternates

```
\dynkin[%
label macro/.code={\alpha_{#1}},
label macro*/.code={\gamma_{#1}},
labels={2,...,5,,7},
labels*={1,3,4,5,6}]{A}{7}
```



Commands to label several roots

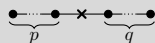
```
\begin{dynkinDiagram}{A}{7}
\dynkinLabelRoots{,\alpha_2,\alpha_3,\alpha_4,\alpha_5,,\alpha_7}
\dynkinLabelRoots*{a,b,c,d,e,f,g}
\end{dynkinDiagram}
```



6. BRACING ROOTS

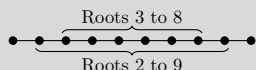
Bracing roots

```
\begin{dynkinDiagram}{A}{*.x*.*}
\dynkinBrace[p]{1}{2}
\dynkinBrace[q]{4}{5}
\end{dynkinDiagram}
```



Bracing roots, and a starred form

```
\begin{dynkinDiagram}{A}{10}
\dynkinBrace[\text{Roots 2 to 9}]{2}{9}
\dynkinBrace*[\text{Roots 3 to 8}]{3}{8}
\end{dynkinDiagram}
```



Bracing roots

```
\newcommand\circleRoot[1]{\draw (root #1) circle (3pt);}
\begin{dynkinDiagram}{A}{**.*.*.*.*.*.*.*}
\circleRoot{4}\circleRoot{7}\circleRoot{10}\circleRoot{13}
\dynkinBrace[y-1]{1}{3}
\dynkinBrace[z-1]{5}{6}
\dynkinBrace[t-1]{11}{12}
\dynkinBrace[x-1]{14}{16}
\end{dynkinDiagram}
```

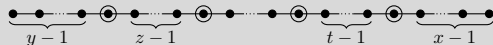
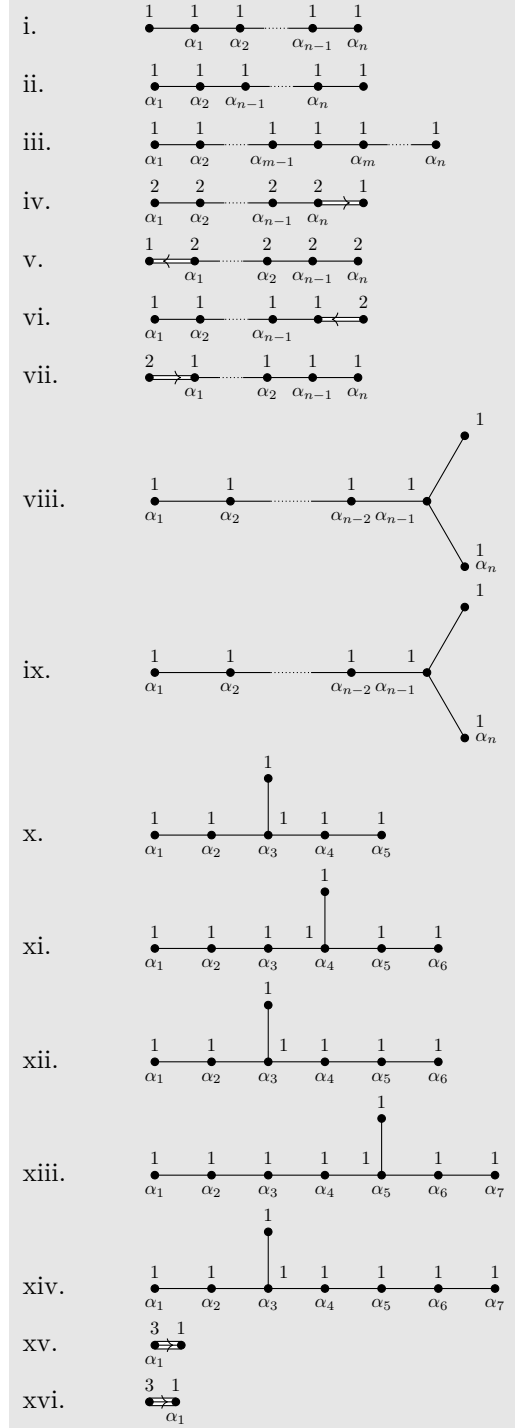
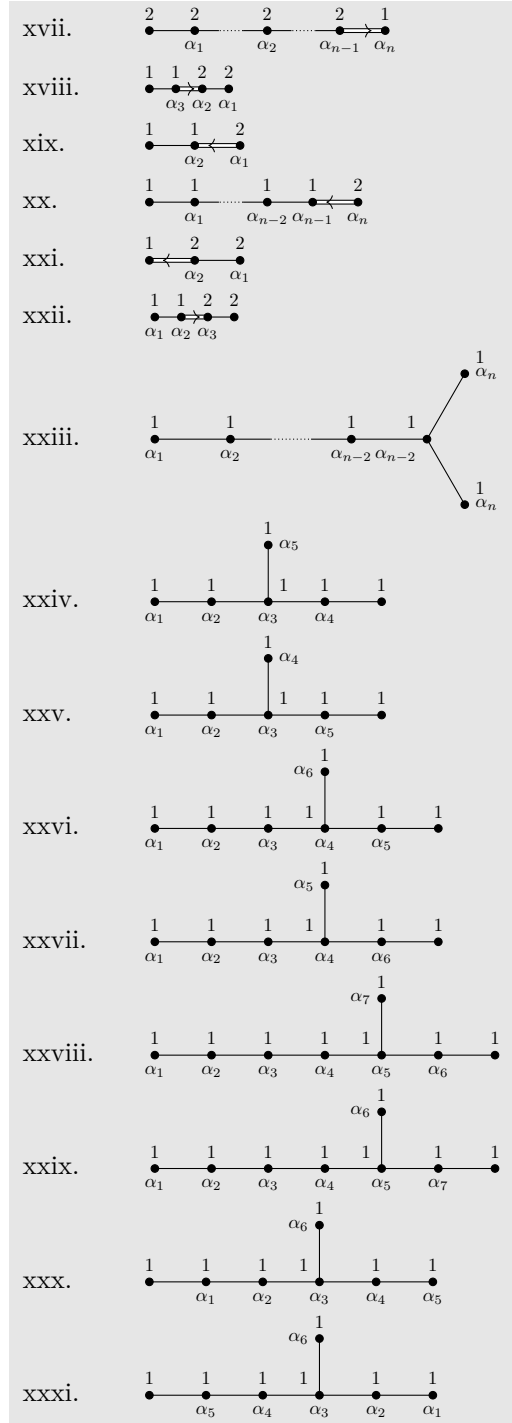


Table 4: Dynkin diagrams from Euler products [17]



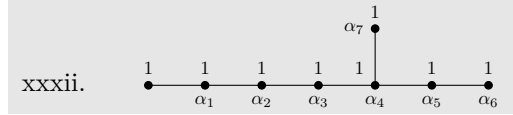
continued ...

Table 4: ...continued



continued ...

Table 4: ...continued



```

\tikzset{/Dynkin diagram,ordering=Dynkin,label macro/.code={\alpha_{#1}}}
\newcounter{EPNo}
\setcounter{EPNo}{0}
\NewDocumentCommand\EP{smmmm}%
{%
\stepcounter{EPNo}\roman{EPNo}. &
\def\eL{.6cm}
\IfStrEqCase{#2}%
{%
{D}{\gdef\eL{1cm}}%
{E}{\gdef\eL{.75cm}}%
{F}{\gdef\eL{.35cm}}%
{G}{\gdef\eL{.35cm}}%
}%
\tikzset{/Dynkin diagram,edge length=\eL}
\IfBooleanTF{#1}%
{\dynkin[backwards,labels*={#4},labels={#5}]{#2}{#3}}
{\dynkin[labels*={#4},labels={#5}]{#2}{#3}}
\\
}%
\begin{longtable}{MM}
\caption{Dynkin diagrams from Euler products \cite{Langlands:1967}}\\
\endfirsthead
\caption{\dots continued}\\
\endhead
\multicolumn{2}{c}{continued \dots}\\
\endfoot
\endlastfoot
\EP{A}{***. **}{1,1,1,1,1}{,1,2,n-1,n}
\EP{A}{***. **}{1,1,1,1,1}{1,2,n-1,n}
\EP{A}{**.*. **}{1,1,1,1,1}{1,2,m-1,,m,n}
\EP{B}{**.*. **}{2,2,2,2,1}{1,2,n-1,n}
\EP*{B}{**.*. **}{2,2,2,2,1}{n,n-1,2,1,}
\EP{C}{**.*. **}{1,1,1,1,2}{1,2,n-1,}
\EP*{C}{**.*. **}{1,1,1,1,2}{n,n-1,2,1,}
\EP{D}{**.*. **}{1,1,1,1,1,1}{1,2,n-2,n-1,n}
\EP{D}{**.*. **}{1,1,1,1,1,1}{1,2,n-2,n-1,n}
\EP{E}{6}{1,1,1,1,1,1}{1,...,5}
\EP*{E}{7}{1,1,1,1,1,1,1}{6,...,1}
\EP{E}{7}{1,1,1,1,1,1,1}{1,...,6}
\EP*{E}{8}{1,1,1,1,1,1,1,1}{7,...,1}
\EP{E}{8}{1,1,1,1,1,1,1,1}{1,...,7}
\EP{G}{2}{1,3}{,1}
\EP{G}{2}{1,3}{1}
\EP{B}{**.*. **}{2,2,2,2,1}{,1,2,n-1,n}
\EP{F}{4}{1,1,2,2}{,3,2,1}

```

```

\EP{C}{3}{1,1,2}{2,1}
\EP{C}{**.*}{1,1,1,1,2}{1,n-2,n-1,n}
\EP*{B}{3}{2,2,1}{1,2}
\EP{F}{4}{1,1,2,2}{1,2,3}
\EP{D}{**.*}{1,1,1,1,1,1}{1,2,n-2,n-2,n,n}
\EP{E}{6}{1,1,1,1,1,1}{1,2,3,4,,5}
\EP{E}{6}{1,1,1,1,1,1}{1,2,3,5,,4}
\EP*{E}{7}{1,1,1,1,1,1,1}{5,...,1,6}
\EP*{E}{7}{1,1,1,1,1,1,1}{6,4,3,2,1,5}
\EP*{E}{8}{1,1,1,1,1,1,1,1}{6,...,1,7}
\EP*{E}{8}{1,1,1,1,1,1,1,1}{7,5,4,3,2,1,6}
\EP*{E}{7}{1,1,1,1,1,1,1}{5,...,1,,6}
\EP*{E}{7}{1,1,1,1,1,1,1}{1,...,5,,6}
\EP*{E}{8}{1,1,1,1,1,1,1,1}{6,...,1,,7}
\end{longtable}

```

7. STYLE

Colours

```

\dynkin[
  edge/.style={blue!50,thick},
  */.style=blue!50!red,
  arrow color=red]{F}{4}

```



Edge lengths

The Dynkin diagram of (A_3) is `\dynkin[edge length=1.2,parabolic=3]{A}{3}`

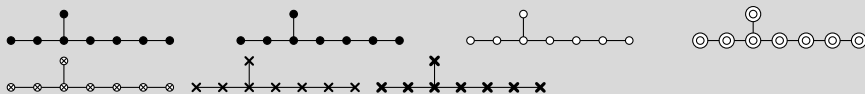
The Dynkin diagram of A_3 is $\times \text{---} \times \text{---} \bullet$

Root marks

```

\dynkin{E}{8}
\dynkin[mark=*]{E}{8}
\dynkin[mark=o]{E}{8}
\dynkin[mark=0]{E}{8}
\dynkin[mark=t]{E}{8}
\dynkin[mark=x]{E}{8}
\dynkin[mark=X]{E}{8}

```




At the moment, you can only use:

- * solid dot
- o hollow circle
- O double hollow circle
- t tensor root
- x crossed root
- X thickly crossed root

Mark styles

The parabolic subgroup $\backslash(E_{8,124})$ is
`\dynkin[parabolic=124,x/.style={brown,very thick}]{E}{8}`

The parabolic subgroup $E_{8,124}$ is 

Sizes of root marks

$\backslash(A_{3,3})$ with big root marks is `\dynkin[root radius=.08cm,parabolic=3]{A}{3}`

$A_{3,3}$ with big root marks is 

8. SUPPRESS OR REVERSE ARROWS

Some diagrams have double or triple edges

`\dynkin{F}{4}`
`\dynkin{G}{2}`



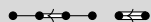
Suppress arrows

`\dynkin[arrows=false]{F}{4}`
`\dynkin[arrows=false]{G}{2}`



Reverse arrows

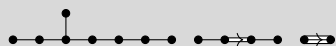
```
\dynkin[reverse arrows]{F}{4}
\dynkin[reverse arrows]{G}{2}
```



9. BACKWARDS AND UPSIDE DOWN

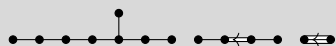
Default

```
\dynkin{E}{8}
\dynkin{F}{4}
\dynkin{G}{2}
```



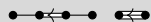
Backwards

```
\dynkin[backwards]{E}{8}
\dynkin[backwards]{F}{4}
\dynkin[backwards]{G}{2}
```



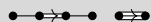
Reverse arrows

```
\dynkin[reverse arrows]{F}{4}
\dynkin[reverse arrows]{G}{2}
```



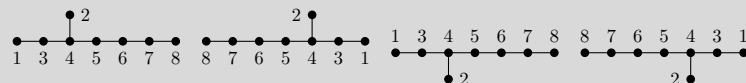
Backwards, reverse arrows

```
\dynkin[backwards,reverse arrows]{F}{4}
\dynkin[backwards,reverse arrows]{G}{2}
```



Backwards versus upside down

```
\dynkin[label]{E}{8}
\dynkin[label,backwards]{E}{8}
\dynkin[label,upside down]{E}{8}
\dynkin[label,backwards,upside down]{E}{8}
```



10. DRAWING ON TOP OF A DYNKIN DIAGRAM

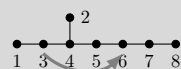
TikZ can access the roots themselves

```
\begin{dynkinDiagram}{A}{4}
  \fill[white,draw=black] (root 2) circle (.15cm);
  \fill[white,draw=black] (root 2) circle (.1cm);
  \draw[black] (root 2) circle (.05cm);
\end{dynkinDiagram}
```



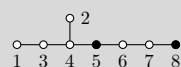
Draw curves between the roots

```
\begin{dynkinDiagram}[label]{E}{8}
  \draw[very thick, black!50,-latex]
    (root 3.south) to [out=-45, in=-135] (root 6.south);
\end{dynkinDiagram}
```



Change marks

```
\begin{dynkinDiagram}[mark=o,label]{E}{8}
  \dynkinRootMark{*}{5}
  \dynkinRootMark{*}{8}
\end{dynkinDiagram}
```

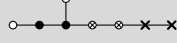


11. MARK LISTS

The package allows a list of root marks instead of a rank:

A mark list

```
\dynkin{E}{oo**ttxx}
```



The mark list `oo**ttxx` has one mark for each root: `o`, `o`, \dots , `x`. Roots are listed in the current default ordering. (Careful: in an affine root system, a mark list will *not* contain a mark for root zero.)

If you need to repeat a mark, you can give a *single digit* positive integer to indicate how many times to repeat it.

A mark list with repetitions

```
\dynkin{A}{x4o3t4}
```

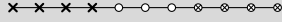


Table 5: Classical Lie superalgebras [10]. We need a slightly larger root radius parameter to distinguish the tensor product symbols from the solid dots.

		<code>\tikzset{/Dynkin diagram,root radius=.07cm}</code>
A_{mn}		<code>\dynkin{A}{o3.oto.oo}</code>
B_{mn}		<code>\dynkin{B}{o3.oto.oo}</code>
B_{0n}		<code>\dynkin{B}{o3.o3.o*}</code>
C_n		<code>\dynkin{C}{too.oto.oo}</code>
D_{mn}		<code>\dynkin{D}{o3.oto.o4}</code>
$D_{21\alpha}$		<code>\dynkin{A}{oto}</code>
F_4		<code>\dynkin{F}{ooot}</code>
G_3		<code>\dynkin[extended,affine mark=t,reverse arrows]{G}{2}</code>

Table 6: Classical Lie superalgebras [10]. Here we see the problem with using the default root radius parameter, which is too small for tensor product symbols.

A_{mn}		<code>\dynkin{A}{o3.oto.oo}</code>
continued ...		

Table 6: ...continued

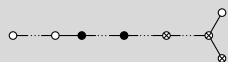
B_{mn}		<code>\dynkin{B}{o3.oto.oo}</code>
B_{0n}		<code>\dynkin{B}{o3.o3.o*}</code>
C_n		<code>\dynkin{C}{too.oto.oo}</code>
D_{mn}		<code>\dynkin{D}{o3.oto.o4}</code>
$D_{21\alpha}$		<code>\dynkin{A}{oto}</code>
F_4		<code>\dynkin{F}{ooot}</code>
G_3		<code>\dynkin[extended,affine mark=t, reverse arrows]{G}{2}</code>

12. INDEFINITE EDGES

An *indefinite edge* is a dashed edge between two roots, $\bullet \cdots \bullet$ indicating that an indefinite number of roots have been omitted from the Dynkin diagram. In between any two entries in a mark list, place a period to indicate an indefinite edge:

Indefinite edges

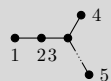
```
\dynkin{D}{o.o*.*.t.to.t}
```



In certain diagrams, roots may have an edge between them even though they are not subsequent in the ordering. For such rare situations, there is an option:

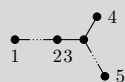
Indefinite edge option

```
\dynkin[make indefinite edge={3-5},label]{D}{5}
```



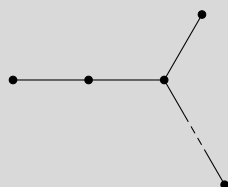
Give a list of edges to become indefinite

```
\dynkin[make indefinite edge/.list={1-2,3-5},label]{D}{5}
```



Indefinite edge style

```
\dynkin[indefinite edge/.style={draw=black,fill=white,thin,densely
dashed},%
edge length=1cm,%
make indefinite edge={3-5}]
{D}{5}
```



The ratio of the lengths of indefinite edges to those of other edges

```
\dynkin[edge length = .5cm,%
indefinite edge ratio=3,%
make indefinite edge={3-5}]
{D}{5}
```

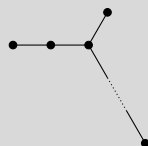


Table 7: Springer’s table of indices [24], pp. 320-321, with one form of E_7 corrected

A_n		
A_n		
B_n		
C_n		
D_n		
E_6		<code>\dynkin{E}{*oooo*}</code>
E_6		<code>\dynkin{E}{o*o*oo}</code>
E_6		<code>\dynkin{E}{o*oooo}</code>

continued ...

Table 7: ...continued

E_6		<code>\dynkin{E}{**ooo*}</code>
E_7		<code>\dynkin{E}{*oooooooo}</code>
E_7		<code>\dynkin{E}{oooooooo*o}</code>
E_7		<code>\dynkin{E}{oooooooo*}</code>
E_7		<code>\dynkin{E}{*oooo*o}</code>
E_7		<code>\dynkin{E}{*oooo**}</code>
E_7		<code>\dynkin{E}{*o**o*o}</code>
E_8		<code>\dynkin{E}{*oooooooo}</code>
E_8		<code>\dynkin{E}{oooooooo*}</code>
E_8		<code>\dynkin{E}{*oooooooo*}</code>
E_8		<code>\dynkin{E}{oooooooo**}</code>
E_8		<code>\dynkin{E}{*oooo***}</code>
F_4		<code>\dynkin{F}{ooo*}</code>
D_4		<code>\dynkin{D}{o*oo}</code>

13. PARABOLIC SUBGROUPS

Each set of roots is assigned a number, with each binary digit zero or one to say whether the corresponding root is crossed or not:

The flag variety of pointed lines in projective 3-space is associated to the Dynkin diagram `\dynkin[parabolic=3]{A}{3}`.

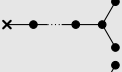
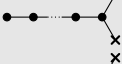
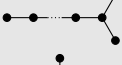
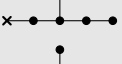
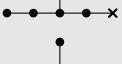
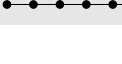
The flag variety of pointed lines in projective 3-space is associated to the Dynkin diagram $\times \rightarrow \bullet$.

Table 8: The Hermitian symmetric spaces

A_n		Grassmannian of k -planes in \mathbb{C}^{n+1}
B_n		$(2n-1)$ -dimensional hyperquadric, i.e. the variety of null lines in \mathbb{C}^{2n+1}
C_n		space of Lagrangian n -planes in \mathbb{C}^{2n}

Table 8: continued ...

Table 8: ...continued

D_n		$(2n - 2)$ -dimensional hyperquadric, i.e. the variety of null lines in \mathbb{C}^{2n}
D_n		one component of the variety of maximal dimension null subspaces of \mathbb{C}^{2n}
D_n		the other component
E_6		complexified octave projective plane
E_6		its dual plane
E_7		the space of null octave 3-planes in octave 6-space

```

\NewDocumentCommand\HSS{mommm}
{#1&\IfNoValueTF{#2}{\dynkin{#3}{#4}}{\dynkin[parabolic=#2]{#3}{#4}}&#5\\}
\renewcommand*{\arraystretch}{1.5}
\begin{longtable}
{>{\columncolor[gray]{.9}}>$1<$>{\columncolor[gray]{.9}}>$1<$>{\columncolor[gray]{.9}}1}
\caption{The Hermitian symmetric spaces}\endfirsthead
\caption{\dots continued}\\ \endhead
\caption{continued \dots}\\ \endfoot
\endlastfoot
\HSS{A_n}{A}{**.*x**.*}{Grassmannian of $k$-planes in $\mathbb{C}^{n+1}$}
\HSS{B_n}[1]{B}{-}{$(2n-1)$-dimensional hyperquadric, i.e. the variety of null lines in $\mathbb{C}^{2n+1}$}
\HSS{C_n}[16]{C}{-}{space of Lagrangian $n$-planes in $\mathbb{C}^{2n}$}
\HSS{D_n}[1]{D}{-}{$(2n-2)$-dimensional hyperquadric, i.e. the variety of null lines in $\mathbb{C}^{2n}$}
\HSS{D_n}[32]{D}{-}{one component of the variety of maximal dimension null subspaces of $\mathbb{C}^{2n}$}
\HSS{D_n}[16]{D}{-}{the other component}
\HSS{E_6}[1]{E}{6}{complexified octave projective plane}
\HSS{E_6}[32]{E}{6}{its dual plane}
\HSS{E_7}[64]{E}{7}{the space of null octave 3-planes in octave 6-space}
\end{longtable}

```

Folded parabolics look bad (zoom in on a root)

```

\dynkin[fold,parabolic=3]{C}{2}
\dynkin[fold,parabolic=3]{G}{2}

```



Folded parabolics: you can try using thicker crosses

```

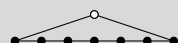
\dynkin[fold,x/.style={very thick,line cap=round},parabolic=3]{C}{2}
\dynkin[fold,x/.style={ultra thick,line cap=round},parabolic=3]{G}{2}

```



14. EXTENDED DYNKIN DIAGRAMS

Extended Dynkin diagrams

`\dynkin[extended]{A}{7}`

The extended Dynkin diagrams are also described in the notation of Kac [15] p. 55 as affine untwisted Dynkin diagrams: we extend `\dynkin{A}{7}` to become `\dynkin{A}[1]{7}`:

Extended Dynkin diagrams

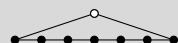
`\dynkin{A}[1]{7}`

Table 9: The Dynkin diagrams of the extended simple root systems

A_1^1		<code>\dynkin[extended]{A}{1}</code>
A_n^1		<code>\dynkin[extended]{A}{}</code>
B_n^1		<code>\dynkin[extended]{B}{}</code>
C_n^1		<code>\dynkin[extended]{C}{}</code>
D_n^1		<code>\dynkin[extended]{D}{}</code>
E_6^1		<code>\dynkin[extended]{E}{6}</code>
E_7^1		<code>\dynkin[extended]{E}{7}</code>
E_8^1		<code>\dynkin[extended]{E}{8}</code>
F_4^1		<code>\dynkin[extended]{F}{4}</code>
G_2^1		<code>\dynkin[extended]{G}{2}</code>

15. AFFINE TWISTED AND UNTWISTED DYNKIN DIAGRAMS

The affine Dynkin diagrams are described in the notation of Kac [15] p. 55:

Affine Dynkin diagrams

$\backslash(A^{\{1\}}_7=\backslash\mathrm{dynkin}\{A\}[1]\{7\}, \backslash$
 $E^{\{2\}}_6=\backslash\mathrm{dynkin}\{E\}[2]\{6\}, \backslash$
 $D^{\{3\}}_4=\backslash\mathrm{dynkin}\{D\}[3]\{4\}\backslash$

$$A_7^{(1)} = \begin{array}{c} \circ \\ \diagup \quad \diagdown \\ \bullet \cdots \bullet \end{array}, \quad E_6^{(2)} = \circ \cdots \bullet \rightleftarrows \bullet, \quad D_4^{(3)} = \circ \rightleftarrows \bullet$$

Table 10: The affine Dynkin diagrams

A_1^1	$\rightleftarrows \bullet$	$\backslash\mathrm{dynkin}\{A\}[1]\{1\}$
A_n^1	$\begin{array}{c} \circ \\ \diagup \quad \diagdown \\ \bullet \cdots \bullet \end{array}$	$\backslash\mathrm{dynkin}\{A\}[1]\{\}$
B_n^1	$\begin{array}{c} \circ \\ \diagup \quad \diagdown \\ \bullet \cdots \bullet \rightleftarrows \bullet \end{array}$	$\backslash\mathrm{dynkin}\{B\}[1]\{\}$
C_n^1	$\begin{array}{c} \circ \\ \diagup \quad \diagdown \\ \bullet \rightleftarrows \bullet \cdots \bullet \rightleftarrows \bullet \end{array}$	$\backslash\mathrm{dynkin}\{C\}[1]\{\}$
D_n^1	$\begin{array}{c} \circ \\ \diagup \quad \diagdown \\ \bullet \cdots \bullet \rightleftarrows \bullet \end{array}$	$\backslash\mathrm{dynkin}\{D\}[1]\{\}$
E_6^1	$\begin{array}{c} \circ \\ \\ \bullet \cdots \bullet \end{array}$	$\backslash\mathrm{dynkin}\{E\}[1]\{6\}$
E_7^1	$\begin{array}{c} \circ \\ \\ \bullet \cdots \bullet \end{array}$	$\backslash\mathrm{dynkin}\{E\}[1]\{7\}$
E_8^1	$\begin{array}{c} \circ \\ \\ \bullet \cdots \bullet \end{array}$	$\backslash\mathrm{dynkin}\{E\}[1]\{8\}$
F_4^1	$\begin{array}{c} \circ \\ \\ \bullet \cdots \bullet \rightleftarrows \bullet \end{array}$	$\backslash\mathrm{dynkin}\{F\}[1]\{4\}$
G_2^1	$\begin{array}{c} \circ \\ \\ \bullet \rightleftarrows \bullet \end{array}$	$\backslash\mathrm{dynkin}\{G\}[1]\{2\}$
A_2^2	$\rightleftarrows \bullet$	$\backslash\mathrm{dynkin}\{A\}[2]\{2\}$
A_{ev}^2	$\begin{array}{c} \circ \\ \\ \bullet \cdots \bullet \rightleftarrows \bullet \end{array}$	$\backslash\mathrm{dynkin}\{A\}[2]\{\mathrm{even}\}$
A_{od}^2	$\begin{array}{c} \circ \\ \\ \bullet \cdots \bullet \rightleftarrows \bullet \end{array}$	$\backslash\mathrm{dynkin}\{A\}[2]\{\mathrm{odd}\}$
D_n^2	$\begin{array}{c} \circ \\ \\ \bullet \cdots \bullet \rightleftarrows \bullet \end{array}$	$\backslash\mathrm{dynkin}\{D\}[2]\{\}$
E_6^2	$\begin{array}{c} \circ \\ \\ \bullet \cdots \bullet \rightleftarrows \bullet \end{array}$	$\backslash\mathrm{dynkin}\{E\}[2]\{6\}$
D_4^3	$\begin{array}{c} \circ \\ \\ \bullet \rightleftarrows \bullet \end{array}$	$\backslash\mathrm{dynkin}\{D\}[3]\{4\}$

Table 11: Some more affine Dynkin diagrams

A_4^2	$\begin{array}{c} \circ \\ \\ \bullet \cdots \bullet \rightleftarrows \bullet \end{array}$	$\backslash\mathrm{dynkin}\{A\}[2]\{4\}$
A_5^2	$\begin{array}{c} \circ \\ \\ \bullet \cdots \bullet \rightleftarrows \bullet \end{array}$	$\backslash\mathrm{dynkin}\{A\}[2]\{5\}$

continued ...

Table 11: ...continued

A_6^2		<code>\dynkin{A}[2]{6}</code>
A_7^2		<code>\dynkin{A}[2]{7}</code>
A_8^2		<code>\dynkin{A}[2]{8}</code>
D_3^2		<code>\dynkin{D}[2]{3}</code>
D_4^2		<code>\dynkin{D}[2]{4}</code>
D_5^2		<code>\dynkin{D}[2]{5}</code>
D_6^2		<code>\dynkin{D}[2]{6}</code>
D_7^2		<code>\dynkin{D}[2]{7}</code>
D_8^2		<code>\dynkin{D}[2]{8}</code>
D_4^3		<code>\dynkin{D}[3]{4}</code>
E_6^2		<code>\dynkin{E}[2]{6}</code>

16. EXTENDED COXETER DIAGRAMS

Extended and Coxeter options together

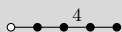



`\dynkin[extended,Coxeter]{F}{4}`

Table 12: The extended (affine) Coxeter diagrams

A_n		<code>\dynkin[extended,Coxeter]{A}{}</code>
B_n		<code>\dynkin[extended,Coxeter]{B}{}</code>
C_n		<code>\dynkin[extended,Coxeter]{C}{}</code>
D_n		<code>\dynkin[extended,Coxeter]{D}{}</code>
E_6		<code>\dynkin[extended,Coxeter]{E}{6}</code>
E_7		<code>\dynkin[extended,Coxeter]{E}{7}</code>
E_8		<code>\dynkin[extended,Coxeter]{E}{8}</code>
F_4		<code>\dynkin[extended,Coxeter]{F}{4}</code>
G_2		<code>\dynkin[extended,Coxeter]{G}{2}</code>

continued ...

Table 12: ...continued

H_3		<code>\dynkin[extended,Coxeter]{H}{3}</code>
H_4		<code>\dynkin[extended,Coxeter]{H}{4}</code>
I_1		<code>\dynkin[extended,Coxeter]{I}{1}</code>

17. KAC STYLE

We include a style called `Kac` which tries to imitate the style of [15].

Kac style	
<code>\dynkin[Kac]{F}{4}</code>	
<hr/>	
	

Table 13: The Dynkin diagrams of the simple root systems in Kac style





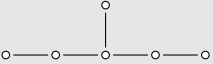
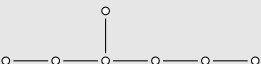
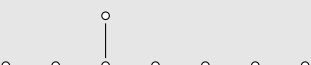



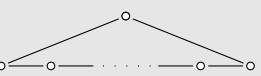
A_n		<code>\dynkin{A}{}</code>
B_n		<code>\dynkin{B}{}</code>
C_n		<code>\dynkin{C}{}</code>
D_n		<code>\dynkin{D}{}</code>
E_6		<code>\dynkin{E}{6}</code>
E_7		<code>\dynkin{E}{7}</code>
E_8		<code>\dynkin{E}{8}</code>
F_4		<code>\dynkin{F}{4}</code>
G_2		<code>\dynkin{G}{2}</code>

Table 14: The Dynkin diagrams of the extended simple root systems in Kac style

A_1^1		<code>\dynkin[extended]{A}{1}</code>
A_n^1		<code>\dynkin[extended]{A}{}</code>

continued ...

Table 14: ...continued

B_n^1		<code>\dynkin[extended]{B}{}</code>
C_n^1		<code>\dynkin[extended]{C}{}</code>
D_n^1		<code>\dynkin[extended]{D}{}</code>
E_6^1		<code>\dynkin[extended]{E}{6}</code>
E_7^1		<code>\dynkin[extended]{E}{7}</code>
E_8^1		<code>\dynkin[extended]{E}{8}</code>
F_4^1		<code>\dynkin[extended]{F}{4}</code>
G_2^1		<code>\dynkin[extended]{G}{2}</code>

Table 15: The Dynkin diagrams of the twisted simple root systems in Kac style

A_2^2		<code>\dynkin{A}{2}{2}</code>
A_{ev}^2		<code>\dynkin{A}{2}{even}</code>
A_{od}^2		<code>\dynkin{A}{2}{odd}</code>
D_n^2		<code>\dynkin{D}{2}{}</code>
E_6^2		<code>\dynkin{E}{2}{6}</code>
D_4^3		<code>\dynkin{D}{3}{4}</code>

18. FOLDED DYNKIN DIAGRAMS

The Dynkin diagrams package has limited support for folding Dynkin diagrams.

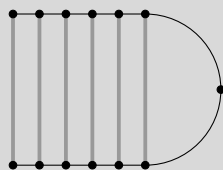
Folding

`\dynkin[fold]{A}{13}`



Big fold radius

```
\dynkin[fold,fold radius=1cm]{A}{13}
```



Small fold radius

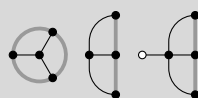
```
\dynkin[fold,fold radius=.2cm]{A}{13}
```



Some Dynkin diagrams have multiple foldings, which we attempt to distinguish (not entirely successfully) by their *ply*: the maximum number of roots folded together. Most diagrams can only allow a 2-ply folding, so `fold` is a synonym for `ply=2`.

3-ply

```
\dynkin[ply=3]{D}{4}
\dynkin[ply=3,fold right]{D}{4}
\dynkin[ply=3]{D}[1]{4}
```



4-ply

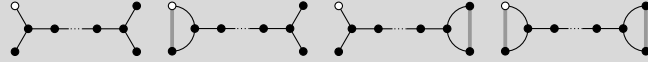
```
\dynkin[ply=4]{D}[1]{4}
```



The $D_\ell^{(1)}$ diagrams can be folded on their left end and separately on their right end:

Left, right and both

```
\dynkin{D}[1]{ } \
\dynkin[fold left]{D}[1]{ } \
\dynkin[fold right]{D}[1]{ } \
\dynkin[fold]{D}[1]{ }
```



We have to be careful about the 4-ply foldings of $D_{2\ell}^{(1)}$, for which we can have two different patterns, so by default, the package only draws as much as it can without distinguishing the two:

Default $D_{2\ell}^{(1)}$ and the two ways to finish it

```
\dynkin[ply=4]{D}[1]{****.*****.*****}%
\
\begin{dynkinDiagram}[ply=4]{D}[1]{****.*****.*****}%
  \dynkinFold[bend right=65]{1}{13}%
  \dynkinFold[bend right=65]{0}{14}%
\end{dynkinDiagram} \
\begin{dynkinDiagram}[ply=4]{D}[1]{****.*****.*****}%
  \dynkinFold{0}{1}%
  \dynkinFold{1}{13}%
  \dynkinFold{13}{14}%
\end{dynkinDiagram}
```

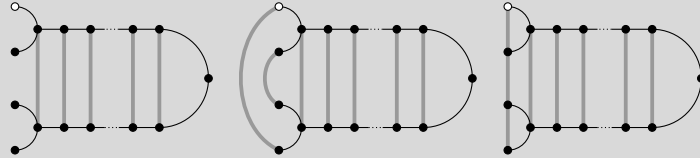


Table 16: Some foldings of Dynkin diagrams. For these diagrams, we want to compare a folding diagram with the diagram that results when we fold it, so it looks best to set `fold radius` and `edge length` to equal lengths.

A_3		<code>\dynkin[fold]{A}[0]{3}</code>
C_2		<code>\dynkin{C}[0]{2}</code>

continued ...

Table 16: ...continued

$A_{2\ell-1}$		<code>\dynkin[fold]{A}{**.*.....**}</code>
C_ℓ		<code>\dynkin{C}{}</code>
B_3		<code>\dynkin[fold]{B}{0}{3}</code>
G_2		<code>\dynkin[reverse arrows]{G}{0}{2}</code>
D_4		<code>\dynkin[ply=3,fold right]{D}{4}</code>
G_2		<code>\dynkin{G}{2}</code>
$D_{\ell+1}$		<code>\dynkin[fold]{D}{}</code>
B_ℓ		<code>\dynkin{B}{}</code>
E_6		<code>\dynkin[fold]{E}{0}{6}</code>
F_4		<code>\dynkin[reverse arrows]{F}{0}{4}</code>
A_3^1		<code>\dynkin[ply=4]{A}{1}{3}</code>
A_1^1		<code>\dynkin{A}{1}{1}</code>
$A_{2\ell-1}^1$		<code>\dynkin[fold]{A}{1}{**.*.....**}</code>
C_ℓ^1		<code>\dynkin{C}{1}{}</code>
B_3^1		<code>\dynkin[ply=3]{B}{1}{3}</code>
A_2^2		<code>\dynkin{A}{2}{2}</code>
B_3^1		<code>\dynkin[ply=2]{B}{1}{3}</code>
G_2^1		<code>\dynkin{G}{1}{2}</code>
B_ℓ^1		<code>\dynkin[fold]{B}{1}{}</code>
D_ℓ^2		<code>\dynkin{D}{2}{}</code>
D_4^1		<code>\dynkin[ply=3]{D}{1}{4}</code>
B_3^1		<code>\dynkin{B}{1}{3}</code>

continued ...

Table 16: ...continued

D_4^1		<code>\dynkin[ply=3]{D}[1]{4}</code>
G_2^1		<code>\dynkin{G}[1]{2}</code>
$D_{\ell+1}^1$		<code>\dynkin[fold]{D}[1]{}</code>
D_ℓ^2		<code>\dynkin{D}[2]{}</code>
$D_{\ell+1}^1$		<code>\dynkin[fold right]{D}[1]{}</code>
B_ℓ^1		<code>\dynkin{B}[1]{}</code>
$D_{2\ell}^1$		<pre> \begin{dynkinDiagram}[ply=4]{D}[1]% {****.*****.*****} \dynkinFold{0}{1} \dynkinFold{1}{13} \dynkinFold{13}{14} \end{dynkinDiagram} </pre>
A_{odd}^2		<code>\dynkin{A}[2]{odd}</code>
$D_{2\ell}^1$		<pre> \begin{dynkinDiagram}[ply=4]{D}[1]% {****.*****.*****} \dynkinFold[bend right=65]{1}{13} \dynkinFold[bend right=65]{0}{14} \end{dynkinDiagram} </pre>
A_{even}^2		<code>\dynkin{A}[2]{even}</code>
E_6^1		<code>\dynkin[fold]{E}[1]{6}</code>
F_4^1		<code>\dynkin[reverse arrows]{F}[1]{4}</code>
E_6^1		<code>\dynkin[ply=3]{E}[1]{6}</code>
D_4^3		<code>\dynkin{D}[3]{4}</code>

continued ...

Table 16: ...continued

E_7^1		<code>\dynkin[fold]{E}[1]{7}</code>
E_6^2		<code>\dynkin{E}[2]{6}</code>
F_4^1		<code>\dynkin[fold]{F}[1]{4}</code>
G_2^1		<code>\dynkin{G}[1]{2}</code>
A_{odd}^2		<code>\dynkin[odd, fold]{A}[2]{****.***}</code>
A_{even}^2		<code>\dynkin{A}[2]{even}</code>
D_3^2		<code>\dynkin[fold]{D}[2]{3}</code>
A_2^2		<code>\dynkin{A}[2]{2}</code>

Table 17: Frobenius fixed point subgroups of finite simple groups of Lie type [4] p. 15

$A_{\ell \geq 1}$		<code>\dynkin{A}{}</code>
${}^2A_{\ell \geq 2}$		<code>\dynkin[fold]{A}{}</code>
$B_{\ell \geq 2}$		<code>\dynkin{B}{}</code>
2B_2		<code>\dynkin[fold]{B}{2}</code>
$C_{\ell \geq 3}$		<code>\dynkin{C}{}</code>
$D_{\ell \geq 4}$		<code>\dynkin{D}{}</code>
${}^2D_{\ell \geq 4}$		<code>\dynkin[fold]{D}{}</code>
3D_4		<code>\dynkin[ply=3]{D}{4}</code>
E_6		<code>\dynkin{E}{6}</code>
2E_6		<code>\dynkin[fold]{E}{6}</code>
E_7		<code>\dynkin{E}{7}</code>
E_8		<code>\dynkin{E}{8}</code>
F_4		<code>\dynkin{F}{4}</code>
2F_4		<code>\dynkin[fold]{F}{4}</code>

continued ...

Table 17: ...continued

G_2		<code>\dynkin{G}{2}</code>
2G_2		<code>\dynkin[fold]{G}{2}</code>

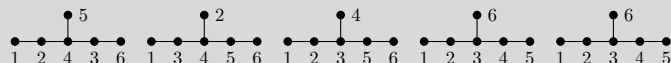
19. ROOT ORDERING

Root ordering

```

\dynkin[label,ordering=Adams]{E}{6}
\dynkin[label,ordering=Bourbaki]{E}{6}
\dynkin[label,ordering=Carter]{E}{6}
\dynkin[label,ordering=Dynkin]{E}{6}
\dynkin[label,ordering=Kac]{E}{6}

```



Default is Bourbaki. Sources are Adams [1] p. 56–57, Bourbaki [3] p. pp. 265–290 plates I-IX, Carter [5] p. 540–609, Dynkin [8], Kac [15] p. 43.

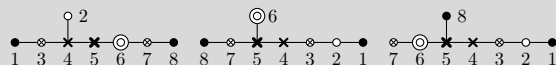
	Adams	Bourbaki	Carter	Dynkin	Kac
E_6					
E_7					
E_8					
F_4					
G_2					

The marks are set down in order according to the current root ordering:

```

\dynkin[label]{E}{*otxX0t*}
\dynkin[label,ordering=Carter]{E}{*otxX0t*}
\dynkin[label,ordering=Kac]{E}{*otxX0t*}

```

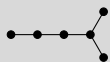


20. CONNECTING DYNKIN DIAGRAMS

We can make some sophisticated folded diagrams by drawing multiple diagrams, each with a name:

Name a diagram

```
\dynkin[name=Bob]{D}{6}
```



We can then connect the two with folding edges:

Connect diagrams

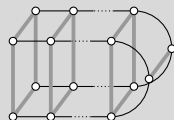
```
\begin{dynkinDiagram}[name=upper]{A}{3}
  \node (current) at ($(\text{upper root 1})+(0,-.3\text{cm})$) {};
  \dynkin[at=(current),name=lower]{A}{3}
  \begin{scope}[on background layer]
    \foreach \i in {1,...,3}%
    {%
      \draw[/Dynkin diagram/fold style]
        ($(\text{upper root } \i)$)
        -- ($(\text{lower root } \i)$);%
    }%
  \end{scope}
\end{dynkinDiagram}
```



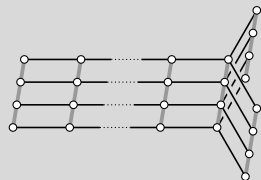
The following diagrams arise in the Satake diagrams of the pseudo-Riemannian symmetric spaces [2].

```
\pgfkeys{/Dynkin diagram,edge length=.5cm,fold radius=.5cm}
\begin{tikzpicture}
  \dynkin[name=1]{A}{IIIb}
  \node (a) at (-.3,-.4){};
  \dynkin[name=2,at=(a)]{A}{IIIb}
  \begin{scope}[on background layer]
    \foreach \i in {1,...,7}%
    {%
      \draw[/Dynkin diagram/fold style]
        ($(\text{1 root } \i)$)
        --
        ($(\text{2 root } \i)$);%
    }%
  \end{scope}
\end{tikzpicture}
```

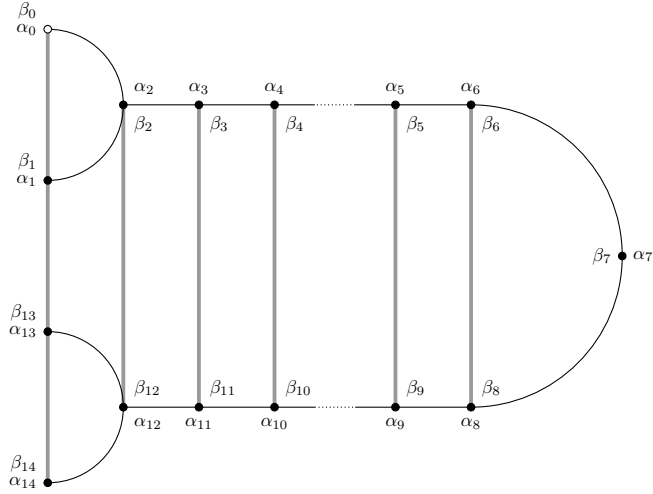
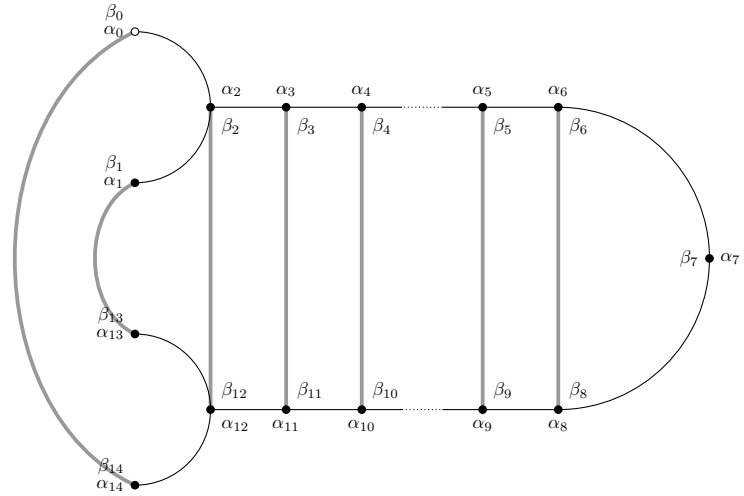
```
\end{tikzpicture}
```



```
\pgfkeys{/Dynkin diagram,
edge length=.75cm,
edge/.style={draw=example-color,double=black,very thick}}
\begin{tikzpicture}
  \foreach \d in {1,...,4}
  {
    \node (current) at ($(\d*.05,\d*.3)$){};
    \dynkin[name=\d,at=(current)]{D}{oo.oooo}
  }
  \begin{scope}[on background layer]
    \foreach \i in {1,...,6}%
    {%
      \draw[/Dynkin diagram/fold style] ($(1 root
\i)$) -- ($(2 root \i)$);%
      \draw[/Dynkin diagram/fold style] ($(2 root
\i)$) -- ($(3 root \i)$);%
      \draw[/Dynkin diagram/fold style] ($(3 root
\i)$) -- ($(4 root \i)$);%
    }%
  \end{scope}
\end{tikzpicture}
```



21. OTHER EXAMPLES

 1D_4 4-ply tied straight: 1D_4 4-ply tied bending:

```

\tikzset{/Dynkin diagram,edge length=1cm,fold radius=1cm}
\tikzset{/Dynkin diagram,label macro/.code={\alpha_{#1}},label macro*/.code={\beta_{#1}}}
\({}^1D_4\) 4-ply tied straight:
\begin{dynkinDiagram}[ply=4]{D}[1]%
{****.****.****}
\dynkinFold{0}{1}
\dynkinFold{1}{13}
\dynkinFold{13}{14}
\dynkinLabelRoots{0,...,14}
\dynkinLabelRoots*{0,...,14}
\end{dynkinDiagram}
\({}^1D_4\) 4-ply tied bending:
\begin{dynkinDiagram}[ply=4]{D}[1]%
{****.****.****}
\dynkinFold[bend right=65]{1}{13}
\dynkinFold[bend right=65]{0}{14}
\dynkinLabelRoots{0,...,14}

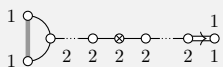
```

```
\dynkinLabelRoots*{0,...,14}
\end{dynkinDiagram}
```

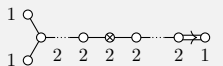
Below we draw the Vogan diagrams of some affine Lie superalgebras [21, 20].

$\mathfrak{sl}(2m|2n)^{(2)}$

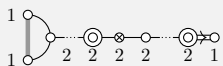
```
\begin{dynkinDiagram}[ply=2,label]{B}[1]{oo.oto.oo}
\dynkinLabelRoot*{7}{1}
\end{dynkinDiagram}
```



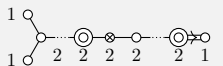
```
\dynkin[label]{B}[1]{oo.oto.oo}
```



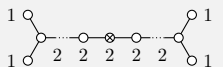
```
\dynkin[ply=2,label]{B}[1]{oo.Oto.Oo}
```



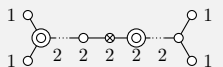
```
\dynkin[label]{B}[1]{oo.Oto.Oo}
```

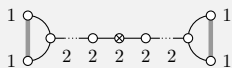


```
\dynkin[label]{D}[1]{oo.oto.ooo}
```

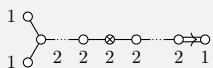


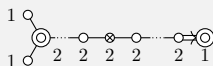
```
\dynkin[label]{D}[1]{oO.otO.ooo}
```

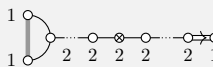


$$\backslash\mathrm{dynkin}[\mathrm{label},\mathrm{fold}]\{\mathrm{D}\}[1]\{\mathrm{oo}.\mathrm{oto}.\mathrm{ooo}\}$$


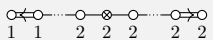
$$\mathfrak{sl}(2m+1|2n)^2$$

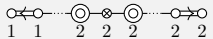
$$\backslash\mathrm{dynkin}[\mathrm{label}]\{\mathrm{B}\}[1]\{\mathrm{oo}.\mathrm{oto}.\mathrm{oo}\}$$


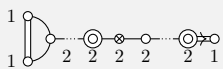
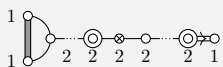
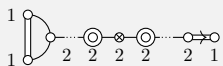
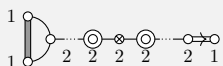
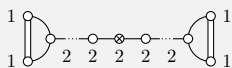
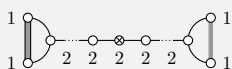
$$\backslash\mathrm{dynkin}[\mathrm{label}]\{\mathrm{B}\}[1]\{\mathrm{o0}.\mathrm{oto}.\mathrm{o0}\}$$


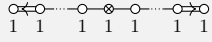
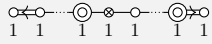
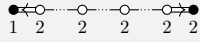
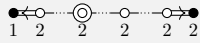
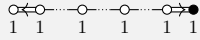
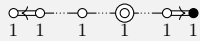
$$\backslash\mathrm{dynkin}[\mathrm{label},\mathrm{fold}]\{\mathrm{B}\}[1]\{\mathrm{oo}.\mathrm{oto}.\mathrm{oo}\}$$


$$\mathfrak{sl}(2m+1|2n+1)^2$$

$$\backslash\mathrm{dynkin}[\mathrm{label}]\{\mathrm{D}\}[2]\{\mathrm{o}.\mathrm{oto}.\mathrm{oo}\}$$


$$\backslash\mathrm{dynkin}[\mathrm{label}]\{\mathrm{D}\}[2]\{\mathrm{o}.\mathrm{0t0}.\mathrm{oo}\}$$


$\mathfrak{sl}(2|2n+1)^{(2)}$
`\dynkin[ply=2,label,double edges]{B}[1]{oo.0to.0o}`

`\dynkin[ply=2,label,double fold]{B}[1]{oo.0to.0o}`

`\dynkin[ply=2,label,double edges]{B}[1]{oo.0t0.oo}`

`\dynkin[ply=2,label,double fold]{B}[1]{oo.0t0.oo}`

 $\mathfrak{sl}(2|2n)^{(2)}$
`\dynkin[ply=2,label,double edges]{D}[1]{oo.ot0.ooo}`

`\dynkin[ply=2,label,double fold
left]{D}[1]{oo.ot0.ooo}`


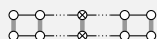
$\mathfrak{osp}(2m|2n)^{(2)}$
`\dynkin[label,label macro/.code={1}]{D}[2]{o.o.to.oo}`

`\dynkin[label,label macro/.code={1}]{D}[2]{o.0.to.0o}`

 $\mathfrak{osp}(2|2n)^{(2)}$
`\dynkin[label,label macro/.code=\lablIt{#1},
affine mark=*]
{D}[2]{o.o.o.o*}`

`\dynkin[label,label macro/.code=\lablIt{#1},
affine mark=*]
{D}[2]{o.0.o.o*}`

 $\mathfrak{sl}(1|2n+1)^4$
`\dynkin[label,label macro/.code={1}]{D}[2]{o.o.o.o*}`

`\dynkin[label,label macro/.code={1}]{D}[2]{o.o.0.o*}`


A^1

```

\begin{tikzpicture}
  \dynkin[name=upper]{A}{oo.t.oo}
  \node (Dynkin current) at (upper root 1){};
  \dynkinSouth
  \dynkin[at=(Dynkin
current),name=lower]{A}{oo.t.oo}
  \begin{scope}[on background layer]
    \foreach \i in {1,...,5}{
      \draw[/Dynkin diagram/fold style]
        ($(\upper root \i)$) --
        ($(\lower root \i)$);
    }
  \end{scope}
\end{tikzpicture}

```



```

\dynkin[fold]{A}[1]{oo.t.oooo.t.oo}

```



```

\dynkin[fold,affine mark=t]{A}[1]{oo.o.ootoo.o.oo}

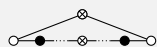
```



```

\dynkin[affine mark=t]{A}[1]{o*.t.*o}

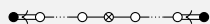
```

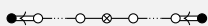
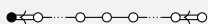
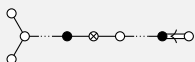
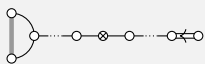
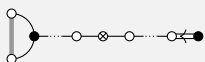
 B^1

```

\dynkin[affine mark=*]{A}[2]{o.oto.o*}

```



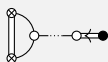
$\backslash\text{dynkin}[\text{affine mark}=\ast]\{A\}[2]\{o.oto.o\ast\}$

 $\backslash\text{dynkin}[\text{affine mark}=\ast]\{A\}[2]\{o.ooo.oo\}$

 $\backslash\text{dynkin}[\text{odd}]\{A\}[2]\{oo.\ast to.\ast o\}$

 $\backslash\text{dynkin}[\text{odd, fold}]\{A\}[2]\{oo.oto.oo\}$

 $\backslash\text{dynkin}[\text{odd, fold}]\{A\}[2]\{o\ast.oto.o\ast\}$

 D^1
 $\backslash\text{dynkin}\{D\}\{otoo\}$

 $\backslash\text{dynkin}\{D\}\{ot\ast o\}$

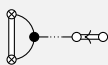

```
\dynkin[fold]{D}{otoo}
```


 C^1

```
\dynkin[double edges,fold,affine
mark=t,odd]{A}[2]{to.o*}
```



```
\dynkin[double edges,fold,affine
mark=t,odd]{A}[2]{t*.oo}
```

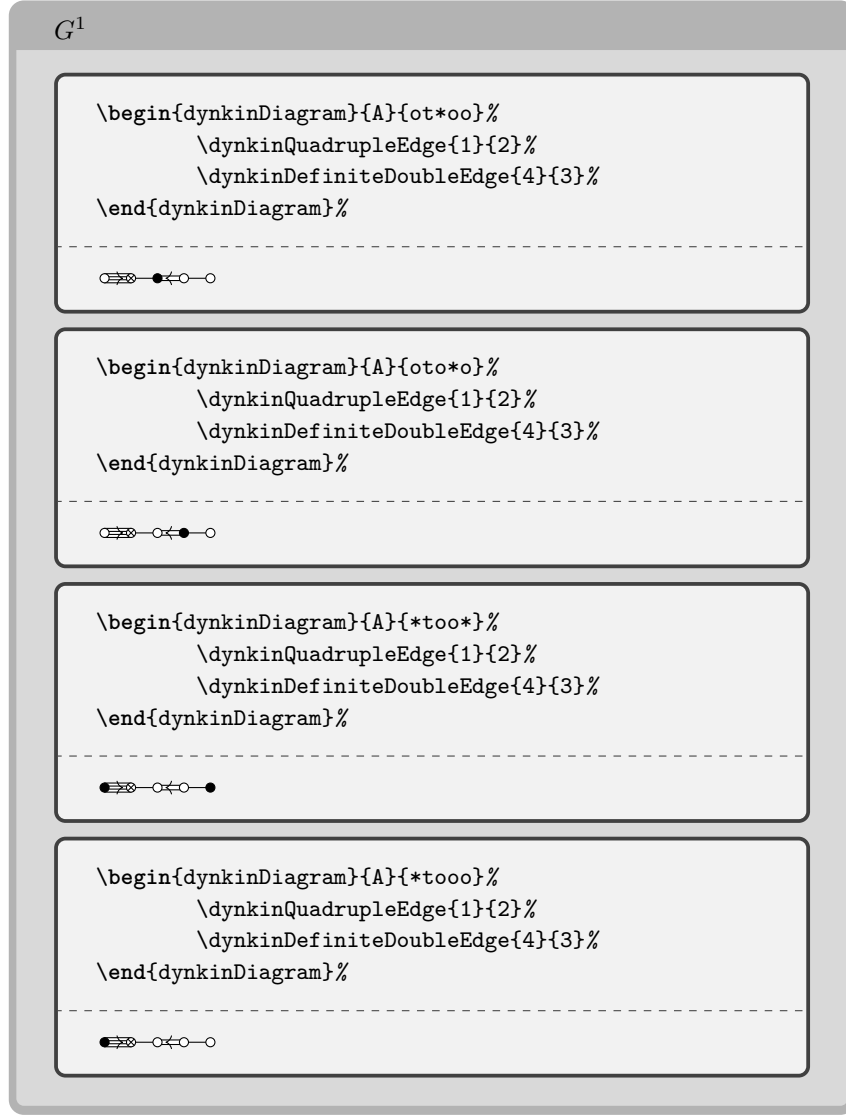

 F^1

```
\begin{dynkinDiagram}{A}{oto*}%
\dynkinQuadrupleEdge{1}{2}%
\dynkinTripleEdge{4}{3}%
\end{dynkinDiagram}%
```



```
\begin{dynkinDiagram}{A}{*too}%
\dynkinQuadrupleEdge{1}{2}%
\dynkinTripleEdge{4}{3}%
\end{dynkinDiagram}%
```





\mathfrak{g}	Diagram	Weights	Roots	Simple roots
A_n		$\frac{1}{n+1}\mathbb{Z}^{n+1} / \langle \sum e_j \rangle$	$e_i - e_j$	$e_i - e_{i+1}$
B_n		$\frac{1}{2}\mathbb{Z}^n$	$\pm e_i, \pm e_i \pm e_j, i \neq j$	$e_i - e_{i+1}, e_n$
C_n		\mathbb{Z}^n	$\pm 2e_i, \pm e_i \pm e_j, i \neq j$	$e_i - e_{i+1}, 2e_n$
D_n		$\frac{1}{2}\mathbb{Z}^n$	$\pm e_i \pm e_j, i \neq j$	$e_i - e_{i+1}, \quad i \leq n-1$ $e_{n-1} + e_n$

g	Diagram	Weights	Roots	Simple roots
E_8		$\frac{1}{2}\mathbb{Z}^8$	$\pm 2e_i \pm 2e_j, \quad i \neq j,$ $\sum_i (-1)^{m_i} e_i, \quad \sum m_i \text{ even}$	$2e_1 - 2e_2,$ $2e_2 - 2e_3,$ $2e_3 - 2e_4,$ $2e_4 - 2e_5,$ $2e_5 - 2e_6,$ $2e_6 + 2e_7,$ $-\sum e_j,$ $2e_6 - 2e_7$
E_7		$\frac{1}{2}\mathbb{Z}^8 / \langle e_1 - e_2 \rangle$	quotient of E_8	quotient of E_8
E_6		$\frac{1}{3}\mathbb{Z}^8 / \langle e_1 - e_2, e_2 - e_3 \rangle$	quotient of E_8	quotient of E_8
F_4		\mathbb{Z}^4	$\pm 2e_i,$ $\pm 2e_i \pm 2e_j, \quad i \neq j,$ $\pm e_1 \pm e_2 \pm e_3 \pm e_4$	$2e_2 - 2e_3,$ $2e_3 - 2e_4,$ $2e_4,$ $e_1 - e_2 - e_3 - e_4$
G_2		$\mathbb{Z}^3 / \langle \sum e_j \rangle$	$\pm(1, -1, 0),$ $\pm(-1, 0, 1),$ $\pm(0, -1, 1),$ $\pm(2, -1, -1),$ $\pm(1, -2, 1),$ $\pm(-1, -1, 2)$	$(-1, 0, 1),$ $(2, -1, -1)$

```

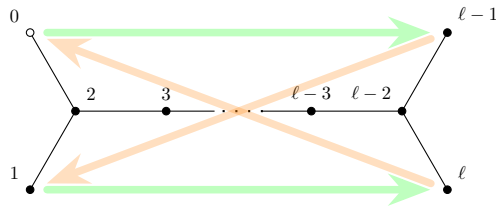
\NewDocumentEnvironment{bunch}{}%
{\renewcommand*{\arraystretch}{1}\begin{array}{@{}l@{}}\midrule{\midrule\end{array}}
\small
\NewDocumentCommand\nct{mm}{\newcolumntype{#1}{\columncolor[gray]{.9}}>{\$}m{\$2cm}<{\$}}
\nct{G}{.3}\nct{D}{2.1}\nct{W}{3}\nct{R}{3.7}\nct{S}{3}
\NewDocumentCommand\LieG{\mathfrak{g}}
\NewDocumentCommand\W{om}{\ensuremath{\mathbb{Z}}^{\#2}\IfValueT{#1}{/\left<#1\right>}}
\renewcommand*{\arraystretch}{1.5}
\NewDocumentCommand\quo{\text{quotient of } E_8}
\begin{longtable}{@{}GDWRS@{}}
\LieG&\text{Diagram}&\text{Weights}&\text{Roots}&\text{Simple roots}\midrule\endfirsthead
\LieG&\text{Diagram}&\text{Weights}&\text{Roots}&\text{Simple roots}\midrule\endhead
A_n&\dynkin{A}{\frac{1}{r+1}\W{n}&\pm e_i, \pm e_i \pm e_j, i \neq j&\pm e_i - e_{i+1}, e_n\\
B_n&\dynkin{B}{\frac{1}{2}\W{n}&\pm e_i, \pm e_i \pm e_j, i \neq j&\pm e_i - e_{i+1}, 2e_n\\
C_n&\dynkin{C}{\frac{1}{2}\W{n}&\pm e_i, \pm e_i \pm e_j, i \neq j&\pm e_i - e_{i+1}, 2e_n\\
D_n&\dynkin{D}{\frac{1}{2}\W{n}&\pm e_i, \pm e_j, i \neq j&\pm e_i - e_{i+1}, \pm e_{n-1} \pm e_n\\
E_8&\dynkin{E}{8}&\frac{1}{2}\W{8}&\pm 2e_i \pm 2e_j, i \neq j, \sum_i (-1)^{m_i} e_i, \sum m_i \text{ even}\\
\begin{bunch}
2e_1 - 2e_2, 2e_2 - 2e_3, 2e_3 - 2e_4, 2e_4 - 2e_5, 2e_5 - 2e_6, 2e_6 + 2e_7, \\
-\sum e_j, 2e_6 - 2e_7
\end{bunch}
\end{bunch}

```

```

E_7&\dynkin{E}{7}&\frac{1}{2}\W[e_1-e_2]{8}&\quo&\quo\\
E_6&\dynkin{E}{6}&\frac{1}{3}\W[e_1-e_2,e_2-e_3]{8}&\quo&\quo\\
F_4&\dynkin{F}{4}&\W{4}&
\begin{bunch}\pm 2e_i,\,\,\,\pm 2e_i \pm 2e_j, \quad i \ne j,\,\,\,\pm e_1 \pm e_2 \pm e_3 \pm e_4
\end{bunch}&
\begin{bunch}2e_2-2e_3,\,\,2e_3-2e_4,\,\,2e_4,\,\,e_1-e_2-e_3-e_4\end{bunch}\\
G_2&\dynkin{G}{2}&\W[\sum e_j]{3}&
\begin{bunch}
\pm(1,-1,0),\,\,\pm(-1,0,1),\,\,\pm(0,-1,1),\,\,\pm(2,-1,-1),\,\,\pm(1,-2,1),\,\,\pm(-1,-1,2)
\end{bunch}&
\begin{bunch}(-1,0,1),\,\,(2,-1,-1)\end{bunch}
\end{longtable}

```



```

\tikzset{big arrow/.style={
  -Stealth,line cap=round,line width=1mm,
  shorten <=1mm,shorten >=1mm}}
\newcommand\catholic[2]{\draw[big arrow,green!25!white]
(root #1) to (root #2);}
\newcommand\protestant[2]{
\begin{scope}[transparency group, opacity=.25]
\draw[big arrow,orange] (root #1) to (root #2);
\end{scope}}
\begin{dynkinDiagram}[edge length=1.2cm,
indefinite edge/.style={thick,loosely dotted},
labels*={0,1,2,3,\ell-3,\ell-2,\ell-1,\ell}]{D}{1}{
\catholic{0}{6}\catholic{1}{7}
\protestant{7}{0}\protestant{6}{1}
\end{dynkinDiagram}

```

22. SYNTAX

The syntax is `\dynkin[<options>]{<letter>}[<twisted rank>]{<rank>}` where `<letter>` is A, B, C, D, E, F or G, the family of root system for the Dynkin diagram, `<twisted rank>` is 0, 1, 2, 3 (default is 0) representing:

- 0 finite root system
- 1 affine extended root system, i.e. of type ⁽¹⁾
- 2 affine twisted root system of type ⁽²⁾
- 3 affine twisted root system of type ⁽³⁾

and `<rank>` is

- (1) an integer representing the rank or
- (2) blank to represent an indefinite rank or
- (3) the name of a Satake diagram as in section 4.

The environment syntax is `\begin{dynkinDiagram}` followed by the same parameters as `\dynkin`, then various Dynkin diagram and TikZ commands, and then `\end{dynkinDiagram}`.

23. OPTIONS

```

text/.style = {TikZ style data},
default : scale=.7
           Style for any labels on the roots.
name = {string},
default : anonymous
           A name for the Dynkin diagram, with anonymous treated as a
           blank; see section 20.
parabolic = {integer},
default : 0
           A parabolic subgroup with specified integer, where the integer
           is computed as  $n = \sum 2^{i-1}a_i$ ,  $a_i = 0$  or  $1$ , to say that root  $i$  is
           crossed, i.e. a noncompact root.
root radius = {number}cm,
default : .05cm
           size of the dots and of the crosses in the Dynkin diagram
edge length = {number}cm,
default : .35cm
           distance between nodes in the Dynkin diagram
edge/.style = TikZ style data,
default : thin
           style of edges in the Dynkin diagram
mark = {o,0,t,x,X,*},
default : *
           default root mark
affine mark = o,0,t,x,X,*,
default : *
           default root mark for root zero in an affine Dynkin diagram
label = true or false,
           continued ...

```

Table 20: ...continued

`default : false`
 whether to label the roots according to the current labelling scheme.
`label macro = <1-parameter TeX macro>`,
`default : #1`
 the current labelling scheme for roots.
`label macro* = <1-parameter TeX macro>`,
`default : #1`
 the current labelling scheme for alternate roots.
`make indefinite edge = <edge pair i - j or list of such>`,
`default : {}`
 edge pair or list of edge pairs to treat as having indefinitely many
 roots on them.
`indefinite edge ratio = <float>`,
`default : 1.6`
 ratio of indefinite edge lengths to other edge lengths.
`indefinite edge/.style = <TikZ style data>`,
`default : draw=black,fill=white,thin,densely dotted`
 style of the dotted or dashed middle third of each indefinite edge.
`backwards = <true or false>`,
`default : false`
 whether to reverse right to left.
`upside down = <true or false>`,
`default : false`
 whether to reverse up to down.
`arrows = <true or false>`,
`default : true`
 whether to draw the arrows that arise along the edges.
`reverse arrows = <true or false>`,
`default : true`
 whether to reverse the direction of the arrows that arise along the
 edges.
`fold = <true or false>`,
`default : true`
 whether, when drawing Dynkin diagrams, to draw them 2-ply.
`ply = <0,1,2,3,4>`,
`default : 0`
 how many roots get folded together, at most.
`fold left = <true or false>`,
`default : true`
 whether to fold the roots on the left side of a Dynkin diagram.
`fold right = <true or false>`,
`default : true`
 whether to fold the roots on the right side of a Dynkin diagram.
`fold radius = <length>`,
`default : .3cm`

continued ...

Table 20: ...continued

the radius of circular arcs used in curved edges of folded Dynkin diagrams.

```
fold style = ⟨TikZ style data⟩,
default : draw=black!40,fill=none,line width=radius
          when drawing folded diagrams, style for the fold indicators.
*/.style = ⟨TikZ style data⟩,
default : draw=black,fill=black
          style for roots like •
o/.style = ⟨TikZ style data⟩,
default : draw=black,fill=black
          style for roots like ◦
O/.style = ⟨TikZ style data⟩,
default : draw=black,fill=black
          style for roots like ⊙
t/.style = ⟨TikZ style data⟩,
default : draw=black,fill=black
          style for roots like ⊗
x/.style = ⟨TikZ style data⟩,
default : draw=black,line cap=round
          style for roots like ×
X/.style = ⟨TikZ style data⟩,
default : draw=black,thick,line cap=round
          style for roots like ✕
fold left style/.style = ⟨TikZ style data⟩,
default :
          style to override the fold style when folding roots together on the
          left half of a Dynkin diagram
fold right style/.style = ⟨TikZ style data⟩,
default :
          style to override the fold style when folding roots together on the
          right half of a Dynkin diagram
double edges = ⟨⟩,
default : not set
          set to override the fold style when folding roots together in a
          Dynkin diagram, so that the foldings are indicated with double
          edges (like those of an  $F_4$  Dynkin diagram without arrows).
double fold = ⟨⟩,
default : not set
          set to override the fold style when folding roots together in a
          Dynkin diagram, so that the foldings are indicated with double
          edges (like those of an  $F_4$  Dynkin diagram without arrows), but
          filled in solidly.
double left = ⟨⟩,
default : not set
```

continued ...

Table 20: ...continued

set to override the `fold` style when folding roots together at the left side of a Dynkin diagram, so that the foldings are indicated with double edges (like those of an F_4 Dynkin diagram without arrows).

`double fold left = $\langle \rangle$,`
`default : not set`
 set to override the `fold` style when folding roots together at the left side of a Dynkin diagram, so that the foldings are indicated with double edges (like those of an F_4 Dynkin diagram without arrows), but filled in solidly.

`double fold right = $\langle \rangle$,`
`default : not set`
 set to override the `fold` style when folding roots together at the right side of a Dynkin diagram, so that the foldings are indicated with double edges (like those of an F_4 Dynkin diagram without arrows).

`double fold right = $\langle \rangle$,`
`default : not set`
 set to override the `fold` style when folding roots together at the right side of a Dynkin diagram, so that the foldings are indicated with double edges (like those of an F_4 Dynkin diagram without arrows), but filled in solidly.

`arrow color = $\langle \rangle$,`
`default : black`
 set to override the default color for the arrows in nonsimply laced Dynkin diagrams.

`Coxeter = \langle true or false \rangle ,`
`default : false`
 whether to draw a Coxeter diagram, rather than a Dynkin diagram.

`ordering = \langle Adams, Bourbaki, Carter, Dynkin, Kac \rangle ,`
`default : Bourbaki`
 which ordering of the roots to use in exceptional root systems as in section 19.

All other options are passed to TikZ.

24. CHANGES IN THE LATEST VERSION

was is

edgeLength	edge length
radius	root radius
affineMark	affine mark
labelMacro	label macro
makeIndefiniteEdge	make indefinite edge
indefiniteEdgeRatio	indefinite edge ratio
indefiniteEdge	indefinite edge
reverseArrows	reverse arrows
foldLeft	fold left
foldRight	fold right
foldradius	fold radius
foldStyle	fold style
leftFoldStyle	fold left style
rightFoldStyle	fold right style
doubleEdges	double edges
doubleFold	double fold
doubleLeft	double left
doubleLeftFold	double fold left
doubleRight	double right
doubleRightFold	double fold right
arrowColor	arrow color

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